Solving Linear Systems [1]

1. Try the following commands (at the prompt and then press Enter):

   (a) `A = [1.2969, 0.8648; 0.2161, 0.1441]`

   (b) `b1 = [1.2969; 0.2161]`

   (c) `x = A\b1`

   (d) Repeat the process but with a vector $b2$ obtained from $b1$ by rounding off to three decimal places.

   (e) Explain exactly what happened. Why was the first answer so simple? Why do the two answers differ by so much?

2. Try the following commands:

   (a) `B = sym(maple('matrix','2,2','(I,J)->sin(I*J)'))`

   (b) `c = [1; 2]`

   and use `x = B\c` to solve $Bx = c$. Then change the 2's to 3's in the first line, change `c` to `[1; 2; 3]` and try to solve again. Use `x = double(x)` to obtain an approximate numerical value of the solution. Try the command `Bn = double(B)`, then `x = Bn\c`. When would an exact symbolic solution and when would an approximate numerical solution be more useful? For big matrices, which type of computation would be faster?

3. Input the matrix:

   $$C = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$$

   and solve $Cx = d$ with `d1 = [4; 8]` and `d2 = [1; 1]`. Use symbolic and non-symbolic versions of C. Explain the results. Which way gives more information?

4. Prepare a report as follows:

   (a) **Using standard mathematical notation**, write down the results of all the computations, except the symbolic solution to the 3 x 3 system in #2. Do **NOT** hand in a printout.

   (b) **Using complete sentences**, briefly answer all of the questions. This includes giving explanations where requested.

The matrix in 1. is nearly singular, causing the linear system to be very sensitive to perturbations. Students are exposed to both symbolic and numerical solutions. The ideas of no solutions or infinitely many solutions are reinforced.